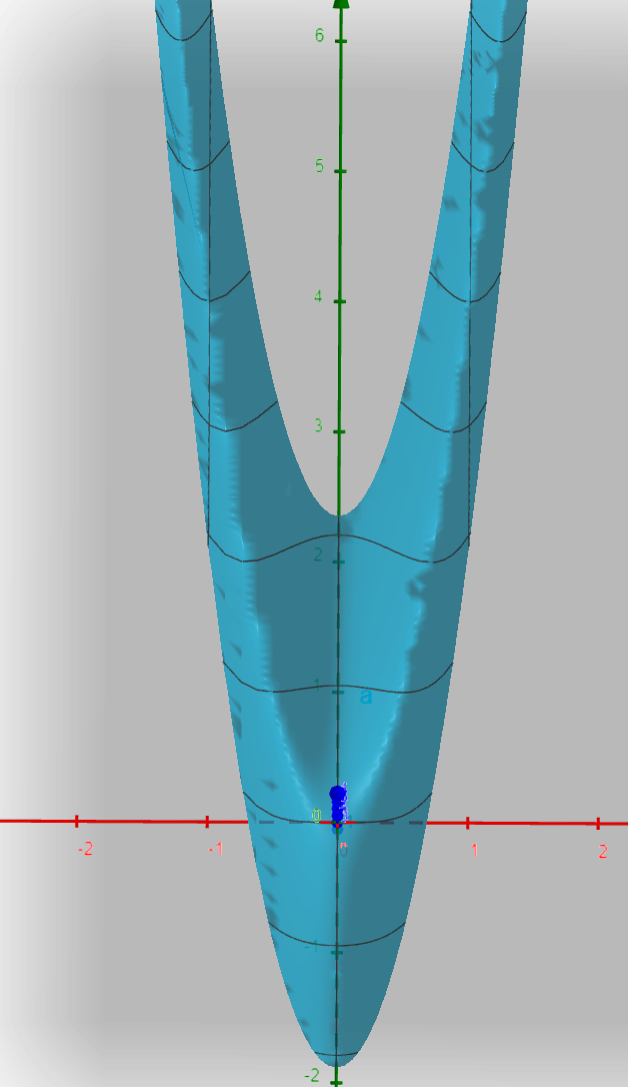
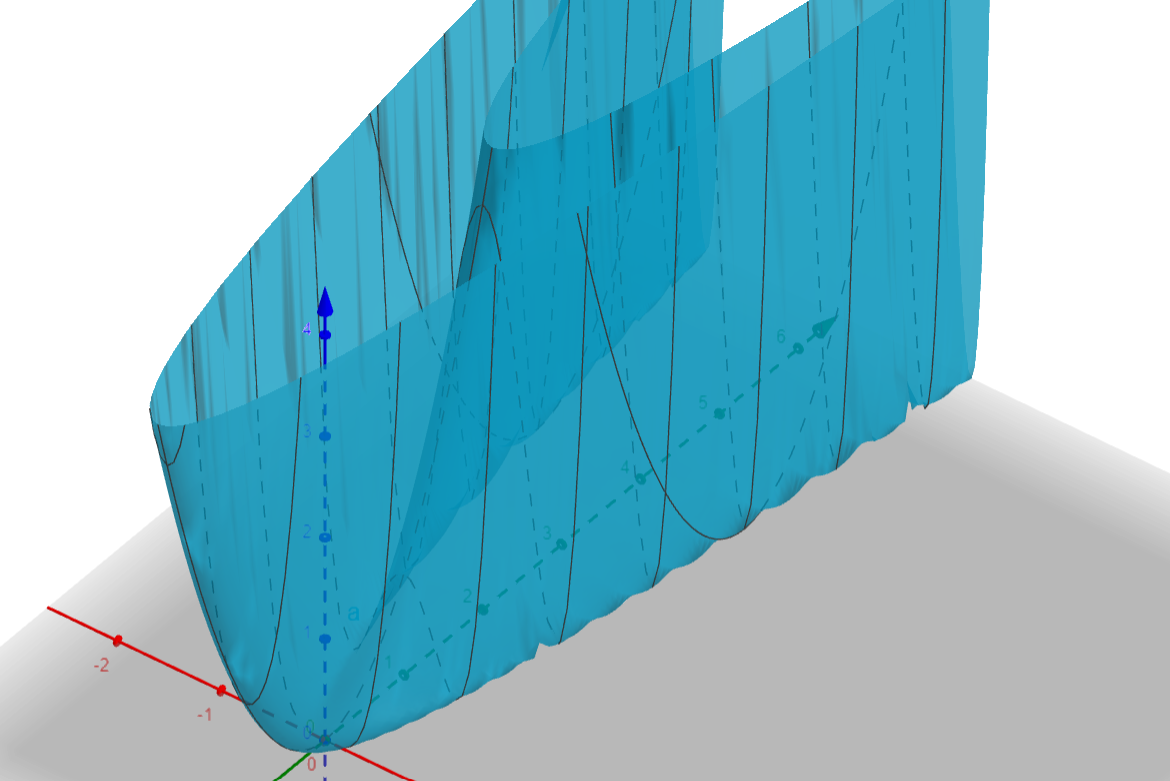
1 Optimality Conditions, Convexity, & One-Dimensional Optimisation

a)

The minimum value for the function is 0 as the term is squared.

.

Hence, all the points for all .



As you can see that all the points on the line are stationary points.

And, they have a global minimum value but not strict local or global

minimum. They are non-strict saddle points.

A picture containing chart

Description automatically generated

If you see closely this can be written as

For the section, we have

Chart, line chart

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So, is a local maximum while (-1, -1) and (1, -1) are local minimums with global minimum value.

For the section, (0, 0) is the global minimum.

This means that and (-1, 0, 0) are strict local minimums with the global minimum value of -1. And (0, 0, 0) may be a saddle point because it is a local maximum in one direction and a minimum in the other two directions.

b) . For what values of is convex?

So, if then will be empty becausewill be negative but cannot be negative as it is the L2 norm. And so, and can never both be zero. And an empty set is convex.

When , then the only point in the set is **a** so it is also convex.

When , then it is a circle as shown below which is convex.

A picture containing bubble chart

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When , then the set is all the points in one half of the region which is also convex.Chart

Description automatically generated with low confidence

When , then the set includes all points except for a circle in the middle so it is not convex.

A picture containing chart

Description automatically generatedTherefore, the set is convex when .

c) One-dimensional function defined on a range with a unique global maximum . If , then , and if , then .

1. Give an example where the Golden Section method might still converge to the global minimum value.

Chart, line chart

Description automatically generated

In this case the algorithm can still converge to the global minimum value at .

1. Give an example where the Golden Section method might not converge to the global minimum value.

Chart, line chart

Description automatically generated

In this case, the global minimum value is at but the Golden Section method will still converge to which is only a local minimum value.